Exercise 1. *Dimers on the $2 \times k$ checker-board*

(1) Show that the number N_k of dimer configurations (complete matchings) on a checker-board of size $2 \times k$ satisfies the Fibonacci relation: $N_k + N_{k+1} = N_{k+2}$. What are N_1, N_2?

(2) Write down N_k using the Kasteleyn determinant: this way we obtain a non-trivial trigonometric recurrence relation.

Exercise 2. *3d Ising Model and dimer configurations*

Consider a 3-dimensional Ising model on a cube divided into N^3 equal size cube spin cells (i.e., each cube is assigned a spin value). For convenience, we will call each cube's six sides top-bottom-left-right-far-near, with opposite sides labelled by opposite words. We impose plus boundary conditions on the top-right-near sides, and minus on the bottom-left-far sides. We would like to study the zero temperature limit $\beta \to \infty$, namely to study the configurations that have non-zero probability in the limit $\beta \to \infty$.

(1) Recall the $2D$ version of this problem from last week's exercise sheet. What is the analogue of the interface between two corners in the $3D$ case? *Hint: If you are having a hard time precisely formulating the surface, describe it as a surface of a 3-dimensional shape, formed by stacking little cubes.*

(2) Lowest energy configurations in the $3D$ Ising model can be mapped one-to-one to hexagonal lattice dimer configurations (with N hexagonal faces on each side; we place dimers on the edges of this lattice to cover all the vertices). To see this, interpret the following picture of Figure 0.1 once as a $3D$ interface between plus and minus spins, and then as a hexagonal lattice dimer tiling (first find the hexagonal lattice hidden in the picture!). Describe how we could draw the Ising interface given a hexagonal lattice dimer tiling.

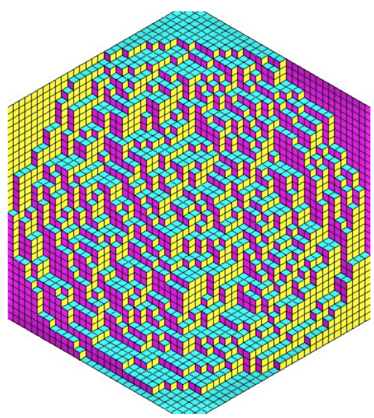

FIGURE 0.1. Richard Kenyon's illustration of a random tiling of a regular hexagon (https://gauss.math.yale.edu/~rwk25/gallery/bppsim.gif)